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The explicit form of the effective action for F1 and D-branes

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ABSTRACT

In this work we consider the full interacting effective actions for fundamental strings and D-branes in arbitrary bosonic type II supergravity backgrounds. The explicit form of these actions is given in terms of component fields, up to second order in the fermions. The results take a compact form exhibiting κ-symmetry, as well as supersymmetry in a background with Killing spinors. Also we give the explicit transformation rules for these symmetries in all cases.

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1 Normal coordinate expansion in M-theory

This work is based on the talk given in the RTN workshop 2003 "The quantum structure of spacetime and the geometric nature of fundamental interactions" in Copenhagen, and contains a summary of the work that appeared in the three papers [1–3]. In these articles we were interested in studying the world-volume theory of various branes of M-theory and in particular on their fermionic sector and supersymmetries. We obtained the corresponding actions in terms of background component fields using the so-called "normal coordinate expansion" [4, 5]. We have worked out these expansions up to second order in the fermionic coordinates. The original strategy used to obtain all D-brane and fundamental string actions [6–9] was to begin with the 11D supermembrane [10] and then, by single dimensional reduction (double dimensional reduction) to 10D, obtain the D2-brane (the fundamental string of type IIA). From here, by means of the correct application of the pertinent form of t-duality rules [11, 12], all the D-branes of type IIA/B (and the fundamental string of type IIB) were found. In the following, the conventions and definitions used can be found in the three articles [1–3]. Nevertheless, we follow mainly the same conventions of [13, 14] for 11D supergravity and [12, 15] for 10D supergravity.

In a superspace formalism, the supercoordinates z^M decompose into bosonic coordinates x^m and fermionic coordinates θ^μ. Here we also introduce a similar decomposition for tangent space vectors y^A, with $A = (a, \alpha)$:

$$\begin{aligned} z^M &= (x^m, \theta^\mu), \\ y^A &= (y^a, y^\alpha). \end{aligned} \tag{1}$$

The normal coordinate expansion is a method based on the definition of normal coordinates in a neighborhood of a given point z^M of superspace. The idea is to parameterize the neighboring points by the tangent vectors along the geodesics joining these points to the origin. Denoting the coordinates at neighboring points by Z^M and the tangent vectors by y^A, we have

$$Z^M = z^M + \Sigma^M(y) \ , \tag{2}$$

where the explicit form of $\Sigma^M(y)$ is found iteratively by solving the geodesic equation. Tensors at the point Z^M may be compared with those at z^M by parallel transport. In this sense, the change in a general tensor under an infinitesimal displacement y^A is

$$\delta T = y^A \nabla_A T \ . \tag{3}$$

Finite transport is obtained by iteration. In this way we may consider the corresponding expansion in the operator δ for any tensor in superspace. For example, consider the vielbein E_M^A

$$E_M{}^A(Z) = E_M{}^A(z) + \delta E_M{}^A(z) + \frac{1}{2}\delta^2 E_M{}^A(z) + \ldots \tag{4}$$

In particular one finds the following fundamental relations by means of which one can expand any tensor iteratively to any order (see for example [16]),

$$\begin{aligned} &\delta y = 0\ , \\ &\delta E^A = \nabla y^A + y^C E^B T_{BC}{}^A\ , \\ &\delta \nabla y^A = -y^B E^C y^D R_{DCB}{}^A\ , \end{aligned} \tag{5}$$

where T is the torsion and R the Riemann tensor.

In our case, since our starting point is the M2-brane, we are interested in an expansion up to second order around a bosonic background of 11D supergravity. We therefore set $z^M = (x^m, 0)$ and $y^A = (0, y^\alpha)$. By means of the 11D superconstraints [13, 14], one finds the following formulas:

$$\begin{aligned} &\delta E^a = 0, \\ &\delta E^\alpha = D y^\alpha + y^\beta e^b T_{b\beta}{}^\alpha\ , \\ &\delta^2 E^a = -i(y^\alpha \Gamma^a{}_{\alpha\beta} D y^\beta + y^\alpha y^\beta T_\beta{}^\gamma \Gamma^a{}_{\gamma\alpha})\ , \\ &\delta^2 E^\alpha = 0\ . \end{aligned} \tag{6}$$

The M2-brane supersymmetric action is,

$$S = -T_{M2} \int d^3\xi \sqrt{\det(-\mathbf{G})} - \frac{T_{M2}}{6} \int d^3\xi \varepsilon^{ijk} \mathbf{A}_{kji}, \tag{7}$$

where $i = (0, 1, 2)$, $T_{M2} = (4\pi^2 l_p^3)^{-1}$, l_p is the 11D Planck length, ($\mathbf{G}$,$\mathbf{A}$) are the pull-back to the supermembrane of the metric and 3-form super-fields of N=1 11D supergravity. At this point, in order to obtain the expanded M2-brane action in component fields, we simply take the M2-brane supersymmetric action, perform substitutions using equations (3,6) and discard terms above second order in fermions. The resulting form of the action is

$$\begin{aligned} S_{M2} = -T_{M2} \int d^3\xi \sqrt{-\det(G)} - \frac{T_{M2}}{6} \int d^3\xi \varepsilon^{ijk} A_{kji} + \\ + \frac{iT_{M2}}{2} \int d^3\xi \sqrt{-G}\, \bar{y}(1 - \Gamma_{M2})\Gamma^i \tilde{D}_i y\ . \end{aligned} \tag{8}$$

where (G,A) are the bosonic part of the corresponding superfields, y is a 11D Majorana spinor, $\Gamma_{M2} = \frac{1}{3!\sqrt{-G}}\epsilon^{ijk}\Gamma_{ijk}$, $\tilde{D}_m = \nabla_m - \frac{1}{288}(\Gamma_m{}^{pqrs} - 8\delta^p_m\Gamma^{qrs})H_{pqrs}$, $H = dA$, $(m, n..)$ are 11D space-time indices and $\tilde{D}_i$ is the pull-back of $\tilde{D}_m$.

κ-symmetry transformation rules relevant for the above action (derived using normal coordinate expansion methods on the superfield form of the κ-symmetry transformation) are

$$\begin{aligned} \delta_\kappa y &= (1+\Gamma_{M2})\kappa + O(y^2) \ , \\ \delta_\kappa x^m &= \frac{i}{2}\bar{y}\Gamma^m(1+\Gamma_{M2})\kappa + O(y^3) \end{aligned} \tag{9}$$

while for supersymmetry transformations we get

$$\begin{aligned} \delta_\varepsilon y &= \varepsilon + O(y^2) \ , \\ \delta_\varepsilon x^m &= -\frac{i}{2}\bar{y}\Gamma^m\varepsilon + O(y^3). \end{aligned} \tag{10}$$

where the spinor ε satisfies the killing spinor equation $\tilde{D}_m\varepsilon = 0$.

2 The fundamental string actions in type IIA/B

The F1 action in type IIA can be obtained from a double dimensional reduction of the M2-brane action (8), then by means of a t-duality transformation, we obtained the corresponding type IIB version. Here, we just show the final form of the actions, although long and tedious calculations were needed in order to obtain the type IIA expression. Also generalizations of the t-duality transformation were introduced to work out the fundamental type IIB string. It is important to say that these actions come in terms of the "natural" operators appearing in the supersymmetric transformation of the supergravity background fields. In the same footing, the well-known symmetries of these actions, like κ-symmetry and supersymmetry, are almost obvious from the expressions that we write here. The form of the actions is:

$$\begin{aligned} S_{(F)} &= -T_{F1}\int d^2\xi\sqrt{-\det(g)} + \tfrac{T_{F1}}{2}\int d^2\xi\epsilon^{ij}b_{ij} + \\ &\quad +iT_{F1}\int d^2\xi\sqrt{-\det(g)}\;\bar{y}\tilde{P}_{(-)}\Gamma^i\tilde{D}_i y, \end{aligned} \tag{11}$$

where for the type IIA case,

$y = (y_+ \ + \ y_-)$ with $\Gamma^{11}y_\pm = \pm y_\pm$ and $P_{(-)} = \frac{1}{2}\left(1 - \frac{1}{2\sqrt{-g}}\epsilon^{ij}\Gamma_{ij}\Gamma^{11}\right)$,

$$
\begin{aligned}
\tilde{D}_m \;&=\; D_m^{(0)} + W_m \;\; , \\
&D_m^{(0)} = \partial_m + \frac{1}{4}\omega_{mab}\Gamma^{ab} + \frac{1}{4\cdot 2!}H_{mab}\Gamma^{ab}\Gamma^{11} \;, \\
&W_m = \frac{1}{8}e^{\phi}\left(\frac{1}{2!}\mathbf{F}^{(2)}_{ab}\Gamma^{ab}\Gamma_m\Gamma^{11} + \frac{1}{4!}\mathbf{F}^{(4)}_{abcd}\Gamma^{abcd}\Gamma_m\right) \;.
\end{aligned} \tag{12}
$$

and for the type IIB case,

$$
y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \quad \text{with} \;\; \Gamma^{11}y_{(1,2)} = y_{(1,2)} \;\; \text{and} \;\; P_{(-)} = \tfrac{1}{2}\left(1 - \tfrac{1}{2\sqrt{-g}}\sigma_3 \otimes \epsilon^{ij}\Gamma_{ij}\Gamma^{11}\right) \;,
$$

$$
\begin{aligned}
\tilde{D}_m \;&=\; \begin{pmatrix} \hat{D}^{(0)}_{(1)m} & 0 \\ 0 & \hat{D}^{(0)}_{(2)m} \end{pmatrix} + \begin{pmatrix} 0 & \hat{W}_{(2)m} \\ \hat{W}_{(1)m} & 0 \end{pmatrix} \;, \\
&\hat{D}^{(0)}_{(1,2)m} = \partial_m + \frac{1}{4}\omega_{mab}\Gamma^{ab} \pm \frac{1}{4\cdot 2!}H_{mab}\Gamma^{ab} \;, \\
&\hat{W}_{(1,2)m} = \mp\frac{1}{8}e^{\phi}\left(\mathbf{F}^{(1)}_a\Gamma^a \pm \frac{1}{3!}\mathbf{F}^{(3)}_{abc}\Gamma^{abc} + \frac{1}{2\cdot 5!}\mathbf{F}^{(5)}_{abcde}\Gamma^{abcde}\right)\Gamma_m \;,
\end{aligned} \tag{13}
$$

where in the expressions for $\hat{D}^{(0)}$ and $\hat{W}$, $\pm$ correspond to the label $(1,2)$. Also in all the above, $(m, n..)$ are 10D space-time indices, g is the 10D metric, $H = db$ and $\mathbf{F}^{(n)}$ are the RR field strength. Note that $\tilde{D}_m$ is precisely the operator appearing in the supersymmetry variation of the 10D gravitino, i.e. $\delta_\epsilon\psi_m = \tilde{D}_m\epsilon$ (see the appendix on [1–3] for the explicit expressions, conventions and definitions).

The corresponding κ-symmetry transformations up to second order in y are

$$
\delta_\kappa y^\alpha = (1+\Gamma_F)\kappa \quad , \quad \delta_\kappa x^m = \frac{i}{2}\bar{y}\Gamma^m(1+\Gamma_F)\kappa \quad , \quad \delta_\kappa A = \delta_\kappa x^m\partial_m A \;, \tag{14}
$$

where $\Gamma_F = \frac{1}{2\sqrt{-g}}\epsilon^{ij}\Gamma_{ij}\Gamma^{11}$ and A is a general field of the supergravity background. The supersymmetry transformations (again up to second order in y) are

$$
\delta_\epsilon y = \epsilon \quad , \quad \delta_\epsilon x^m = -\frac{i}{2}\bar{y}\Gamma^m\kappa \quad , \quad \delta_\epsilon A = \delta_\epsilon x^m\partial_m A \;, \tag{15}
$$

where again A is a general field of the supergravity background and the fermion ϵ is actually a killing spinor of the bosonic background, i.e. $\delta_\epsilon\psi_m = \delta_\epsilon\lambda = 0$, where ψ_m and λ are the gravitino and the dilatino respectively.

3 D-brane actions in type IIA/B

To obtain the general form of the D-brane actions, we performed first a single dimensional reduction from the M2-brane action to 10D to obtain the D2-brane action of type IIA theory. Then by means of t-duality we recovered all the other D-brane actions. Although conceptually the program is clear, it is far from straightforward since the calculation is plague with technical problems and subtleties like for example, the generalization of t-duality rules for the worldvolume fermions and its compatibility with already existent rules for supergravity like those given by Hassan [12].

The resulting actions are

$$\begin{aligned} S_{Dp} &= S_{Dp}^{(0)} + S_{Dp}^{(2)} + O(y^4), \\ S_{Dp}^{(0)} &= -T_{Dp}\int d^3\xi e^{-\phi}\sqrt{-(g+\mathcal{F})} + T_{Dp}\int C\, e^{-\mathcal{F}}\,, \\ S_{Dp}^{(2)} &= \frac{iT_{Dp}}{2}\int d^3\xi e^{-\phi}\sqrt{-(g+\mathcal{F})}\bar{y}(1-\tilde{\Gamma}_{D_p})(\Gamma^i D_i - \Delta + L_p)y. \end{aligned} \quad (16)$$

with

$$\begin{aligned} \tilde{\Gamma}_{D(2n)} &= \frac{1}{\sqrt{-(g+\mathcal{F})}}\sum_{q+r=n}\frac{\epsilon^{i_1\cdots i_{2q}j_1\cdots j_{2r+1}}}{q!2^q(2r+1)!}\mathcal{F}_{i_1i_2}\cdots\mathcal{F}_{i_{2q-1}i_{2q}}\Gamma_{j_1\ldots j_{2r+1}}(\Gamma^{\underline{\varphi}})^{r+1}\,, \\ \tilde{\Gamma}_{D(2n+1)} &= \frac{-i\sigma_2}{\sqrt{-(g+\mathcal{F})}}\sum_{q+r=n+1}\frac{\epsilon^{i_1\cdots i_{2q}j_1\cdots j_{2r}}}{q!2^q(2r)!}\mathcal{F}_{i_1i_2}\cdots\mathcal{F}_{i_{2q-1}i_{2q}}\Gamma_{j_1\ldots j_{2r}}(\hat{\Gamma}^{\underline{\varphi}})^r. \end{aligned} \quad (17)$$

$$\begin{aligned} L_{2n+1} &= \sum_{q\geq 1,q+r=n+1}\frac{\epsilon^{i_1\cdots i_{2q}j_1\cdots j_{2r+1}}(-i\sigma_2)(\hat{\Gamma}^{\underline{\varphi}})^r}{q!2^q(2r+1)!\sqrt{-(g+\mathcal{F})}}\mathcal{F}_{i_1i_2}\cdots\mathcal{F}_{i_{2q-1}i_{2q}}\Gamma_{j_1\ldots j_{2r+1}}{}^k\hat{D}_k\,, \\ L_{2n} &= \sum_{q\geq 1,q+r=n}\frac{\epsilon^{i_1\cdots i_{2q}j_1\cdots j_{2r+1}}(-\Gamma^{\underline{\varphi}})^{r+1}}{q!2^q(2r+1)!\sqrt{-(g+\mathcal{F})}}\mathcal{F}_{i_1i_2}\cdots\mathcal{F}_{i_{2q-1}i_{2q}}\Gamma_{j_1\ldots j_{2r+1}}{}^k D_k\,, \end{aligned} \quad (18)$$

where for the type IIA case

$$\begin{aligned} D_m &= D_m^{(0)} + W_m \\ \Delta &= \Delta^{(1)} + \Delta^{(2)}\,, \end{aligned} \quad (19)$$

with

$$\begin{aligned} D_m^{(0)} &= \partial_m + \frac{1}{4}\omega_{mab}\Gamma^{ab} + \frac{1}{4\cdot 2!}H_{mab}\Gamma^{ab}\Gamma^{\underline{\varphi}} \\ W_m &= \frac{1}{8}e^{\phi}\left(\frac{1}{2!}\mathbf{F}^{(2)}_{ab}\Gamma^{ab}\Gamma_m\Gamma^{\underline{\varphi}} + \frac{1}{4!}\mathbf{F}^{(4)}_{abcd}\Gamma^{abcd}\Gamma_m\right) \end{aligned}$$

$$
\begin{aligned}
\Delta^{(1)} &= \frac{1}{2}\left(\Gamma^m\partial_m\phi + \frac{1}{2\cdot 3!}H_{abc}\Gamma^{abc}\Gamma^{\underline{\varphi}}\right) \\
\Delta^{(2)} &= \frac{1}{8}e^{\phi}\left(\frac{3}{2!}\mathbf{F}^{(2)}_{ab}\Gamma^{ab}\Gamma^{\underline{\varphi}} + \frac{1}{4!}\mathbf{F}^{(4)}_{abcd}\Gamma^{abcd}\right)\ .
\end{aligned}
\tag{20}
$$

and for the type IIB case

$$
\begin{aligned}
\hat{D}_m &= \hat{D}^{(0)}_m + \sigma_1\otimes\hat{W}_m \\
\hat{\Delta} &= \hat{\Delta}^{(1)} + \sigma_1\otimes\hat{\Delta}^{(2)}\ .
\end{aligned}
\tag{21}
$$

with

$$
\begin{aligned}
\hat{D}^{(0)}_{(1,2)m} &= \partial_m + \frac{1}{4}\omega_{mab}\Gamma^{ab} \pm \frac{1}{4\cdot 2!}H_{mab}\Gamma^{ab} \\
\hat{W}_{(1,2)m} &= \frac{1}{8}e^{\phi}\left(\mp\mathbf{F}^{(1)}_a\Gamma^a - \frac{1}{3!}\mathbf{F}^{(3)}_{abc}\Gamma^{abc} \mp \frac{1}{2\cdot 5!}\mathbf{F}^{(5)}_{abcde}\Gamma^{abcde}\right)\Gamma_m \\
\hat{\Delta}^{(1)}_{(1,2)} &= \frac{1}{2}\left(\Gamma^m\partial_m\phi \pm \frac{1}{2\cdot 3!}H_{abc}\Gamma^{abc}\right) \\
\hat{\Delta}^{(2)}_{(1,2)} &= \frac{1}{2}e^{\phi}\left(\pm\mathbf{F}^{(1)}_a\Gamma^a + \frac{1}{2\cdot 3!}\mathbf{F}^{(3)}_{abc}\Gamma^{abc}\right)\ .
\end{aligned}
\tag{22}
$$

where $\Gamma^{11} = \Gamma^{\underline{\phi}}$ and $\hat{\Gamma}^{\underline{\phi}} = \sigma_3\otimes\Gamma^{\underline{\phi}}$.

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